BAKHTINIAN DIALOGUE AND HEGELIAN DIALECTIC IN MATHEMATICAL AND PROFESSIONAL EDUCATION

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We argue that the distinction between dialogue (after Bakhtin) and dialectic (after Hegel, Marx, Vygotsky), that Matusov has previously highlighted, is of key importance to mathematics education. According to Matusov, for Bakhtin these concepts are incommensurable since dialectics implies and the dialogism denies telos (a target). In this essay we argue that mathematical dialogue can and should have teleology Matusov says is implied by dialectics. Thus a good dialogue might involve mathematical (or professional) negation and sublation, providing the dialectic for mathematical (or professional) 'progress' and development. To make this concrete, we illustrate the argument with a lesson study in which progress emerging from dialogues is interpreted in dialectical terms.

INTRODUCTION AND BACKGROUND CONCEPTS

This essay aims to make progress in debates ongoing in the field of socio-cultural theory about dialogism and dialectics, showing how and why this debate is a significant one for mathematics education in particular. In everyday mathematics education terms, we are concerned with dialogue, in mathematics classrooms (between learners and teachers) and in staff rooms (between teachers, for example engaged in lesson study). The concern is whether a dialogue goes beyond the sharing of meanings by those involved, to a point where some sort of progress or development is achieved, for example mathematically, or perhaps professionally. Such developments can be said to have 'telos', a progressive direction.

A case in point is from our lesson study project. Scenario: A group of children are counting the steps (strides) made by Usain Bolt in a video of his Olympic 100 metres winning sprint. The year 2 (6-year-old) children's answers to the question, *How many steps does he make (from the beginning of the video until he crosses the finish line)*?, produce many and varied answers from 18, or 19 (the answer we thought correct) right up to 20s and 30s. In several trials of counting, the children's answers converged somewhat (and gradually), but there were still differences. During the subsequent activity the children modelled the situation physically in the gymnasium, counting the steps laid out for them along number lines across the gym. One of the anticipated issues in children's counting of the steps becomes clear and is discussed as part of the classroom dialogue: should the first foot-print (i.e. that before the start of the race) be counted as zero or one? Another, unanticipated, issue (among others) arises from one of the children counting only up to the last foot-print (before the finishing line) and refusing to cross the line from the 18th to the 19th foot-print (see Figure 1)!

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Now on the one hand progress in the classroom dialogue here might be the result of the learners coming to agree with the teacher's preferred understanding of the situation and the mathematics, for example a convergence on an agreed answer of 19 steps. On the other hand, one might think progress in the dialogue arises due to the teachers/researchers understanding the children's mathematics and their perspectives. Bakhtinian *dialogism* theorises a dialogue as *monologic* if the authority asserts their preferred, 'correct' answer, but as *dialogic* if an internally persuasive discourse is constructed such that both subjectivities have an opportunity to engage through dialogue with the other's discourse (Bakhtin, 1981). Dialogism might involve progress in each person's subjective understanding, and to that extent we will argue would be subjectively dialectical. The question arises, though, whether this subjective progress represents 'objective' telos in mathematics, i.e. can the dialogue be said to be mathematically more advanced, in an objective, historical-cultural sense?

Figure 1 shows the foot-prints (with numbered marks below them from zero before the start to 19 after the finish line) and the foot-steps (shown by arrows).

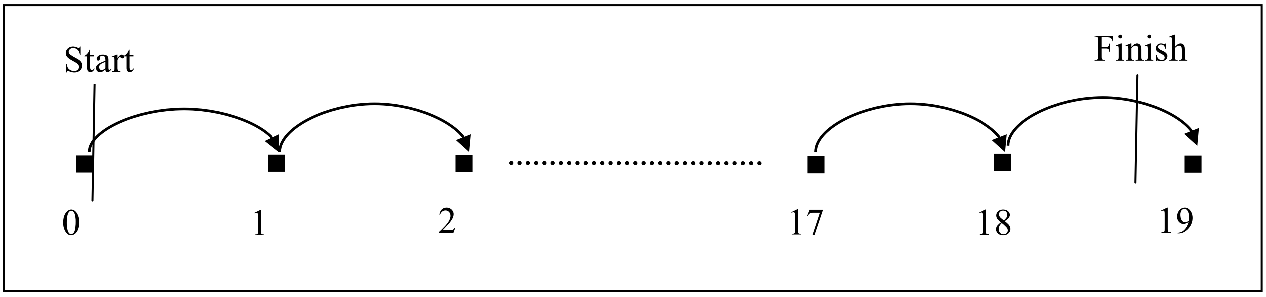

Figure 1: Shows foot-prints and 'arrowed' foot-steps

The simplest answer is that the teachers'/researchers' mathematics – if they are not mistaken in reference to the curriculum (and the curriculum is in turn not mistaken in relation to the cultural-historical state of mathematics) – will be more advanced culturally than the children's mathematics. The telos then is here defined by the definition of the mathematics in the curriculum targeted by the teacher (end of story). Progress is objective if the children (and perhaps the teachers/researchers) make progress towards this target. In this view the dialogue is functional if and only if it allows the correct mathematics to become internally persuasive for the children: this is what Matusov criticises as monologic in Bakhtin's terms.

On the other hand, the constructivist tradition values the children's own mathematics as genuinely mathematical in its own right: their perspective should not be expected to converge with the teachers' or that of the curriculum, negating the above view. Progress is made by the child's mathematics being more (subjectively) adequate in explaining the situation or task, and so on. This is loosely the constructivist position criticised by Radford (2013).

We will now argue that Hegelian dialectics might offer another perspective: in true dialectical terms we seek a new sublation of these two positions.

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THEORETICAL ARGUMENT AND DIALECTICS

This situation is addressed by Matusov, though not in the mathematical context. First he characterises Bakhtinian dialogism as 'intersubjectivity without (necessarily achieving) agreement' (Matusov, 1996), and more recently in characterising Hegelian dialectics as anathema to dialogism, arguing that it imposes a notion of telos that is more akin to monologue (Matusov, 2011).

We agree that the dialectic implies progress, i.e. a teleological process, though the actual end-target may not be anywhere visible during the process itself. Only from the vantage point of history can one see with some surety where progress was made. Nevertheless, the dialectical process that takes a notion (e.g. mathematics, or 'counting') through its negation to its 'sublation' in a new notion, can be regarded as in itself progressive or developmental (for 'notion' here we might also say 'idea' or 'concept'). It is the character of negation and sublation in Hegel (2009) that characterises the type of progress and teleology: We consider now why this is so.

Sublation requires that the notion and its negation are unified in the new (this is often simplistically presented as a 'synthesis' of a thesis and its negation). That is, the original notion and its negation do not disappear, but remain present subsumed within the new form. As such, the sublation represents progress from the old to the new level of thought. In his science of logic, Hegel begins with the notion of 'being' as perhaps the simplest notion of ontology; its negation is non-being or pure 'nothing'. The sublation of being and non-being is then 'becoming', i.e. the movement from nothing to being (and additionally from being to nothing – so dying is also a kind of un-becoming, as unbecoming as it sounds).

Here, as in every dialectic, thought (the notion, idea or concept) and its negation do not quite disappear but are sublated in relation to each other in the new thought. Indeed the concept of sublation (one that Hegel does not explicitly address, see Palm, 2009) implies preservation as well as change: in reality, preservation indeed requires adaptation and change. Sublation is thus a self-contradictory concept in this sense.

One can read the notion of number as developing in this way: the notion of whole number is negated in practice by the fact that not all quantities can be counted as whole numbers (maybe not even 'steps'), and the new number form (maybe fractions or decimals) then sublates the 'whole number' and its practical negation. Whole numbers (and their negation) have not disappeared here, but they are preserved in a new form (thus the whole number 'one' perhaps becomes the rational number 1.0, which is the same and yet not quite the same as the whole number 'one' that was negated).

In this account the concept of practice has been adduced. Hegelian dialectic was originally presented as being the movement of 'pure thought' in itself, and the concept of social practice in this engagement is usually stressed as Marxian. Marx's claim to have turned Hegel dialectics on its head (or feet) is easily misunderstood here: for Marx conscious thought is still a key moment in practice. Thus, in the preceding sublation, the practice of counting in new contexts pushes new thought: the cognitive

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conflict induced provides one contradictory moment (in the moment of thought), and the contradictory positions in the practice and the dialogue provide others (in the objective and intersubjective moments respectively).

Now Marx and Lenin's readings of Hegel and the dialectic become relevant: we recall that Vygotsky (1986) appealed to both in this famous passage in *Thought and Language*, quoting volume 29 of Lenin's collected works as follows:

> Man's practice, repeated a billion times, anchors the figures of logic in his consciousness. These figures have the strength of prejudice, their axiomatic character, precisely (and only) because of this repetition. (p. 198)

Ultimately, for Marx, Lenin and Vygotsky, the dialectic of theory and practice is developmental to the extent that thought proves efficacious in practice. As such, intersubjective dialogue (for example, between learners and teachers, and the curriculum, and the community of mathematicians) provides a powerful contradictory moment for dialectics in discourse, but material practice (including related discourses) is decisive. Dialogue without a practical context that proves a notion cannot be decisively progressive. Thus, Marx used Hegelian dialectics himself in thinking through his analyses of capital and labour in the *Grundrisse*: labour is the 'negation' of capital, and capital in order to conserve-renew itself must be negated through its investment in labour, and through the surplus value capital-and-labour become sublated in new, expanded capital, otherwise it will die in a pre-capitalist form of money in pure circulation (Marx, 1973). Yet it is the fully produced theory in *Das Kapital* that he publishes – in which dialectics are secondary to data and theory.

In the context of measuring Usain Bolt's footsteps, as we researchers thought about the children's answers (was it 18 or 19 steps?), we saw that both were correct answers to different questions, and that the truth might even be that the answer is somewhere between the two, depending on what we choose to mean by the term 'step'. This is perhaps the sublation of the previously constituted truth (the answer we thought was 19) that we achieved through its negation in our joint lesson experience in practice with the children. In the next section we look at the dialogue of teachers discussing this very issue in their lesson study reflections.

DIALECTICAL ANALYSIS OF LESSON STUDY DIALOGUE

The following dialogue took place in the teacher-researcher meeting following the 'Usain Bolt lesson' referred to above. This lesson was part of our lesson study research project (Williams & Ryan, 2013) where we have been working with staff in a primary school developing mathematical dialogue in their classrooms (Reception to year 6 classes: 4/5- to 10/11-year-olds). The lesson had been planned jointly by a core group of teachers and university researchers for year 2 children. A key point for the team's lesson plan was to make the number line 'come alive' for the children, and in particular to address the problem of counting the 'ticks or the jumps' on the number line, which the team agreed is a key mathematical problem (see Ryan & Williams, 2007, pp. 93-94). This particular lesson event involved the entire school staff in the lesson

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observation and the extensive after-lesson discussion. The two themes of the discussion that emerged were: how would we teach this lesson again or what would we do in the follow-up lesson? 'Lesson study' follows the Japanese model of research-led continuing teacher professional development cycles, though we have adapted it for local conditions and focus (Williams, Ryan & Morgan, 2013).

In the following transcript, we see a discussion reflecting on the experience of teaching and observing the children and the way the teachers-researchers imagine developing new teaching based on this experience. This can be read as dialogism: the teachers and researchers are engaged in trying to make sense of the experience and to make sense of each others' meanings for what occurred. But is there progress, and is there a dialectic?

Teacher 1: Could you not give two different answers… And so – it could be this answer, it could be this answer … and actually get the children to be involved in deciding why one answer or the other ... and effectively … those who think it's 18, and those who think it's 19 .. .We're not saying this is the definitive answer, we are just saying it is one that could be explained...

Researcher 1: I think we want to get to what Benny [*pseudonym, one of the children*] said where you count, (from) where the zero is ... and historically that's what humankind … had this problem ... so I'd suggest, yes, how did these people get 18, how 19, … so they're engaged in ... so we get: THEY got 18 because they started counting HERE, that's what we want articulation of ... These people call this foot-print "one", and they call it "zero" ... so once you have that out on the table you ... then you can start a debate. So which...

Teacher 1: If you do that, aren't you going to be giving them (an) answer already? If you are saying this is one or zero, aren't you? Whereas if you ask them the reasons for two answers, you aren't explaining why... it's up to them to prove or disprove...

Researcher 1: Yes, I'm saying HOW did these children get 18, how 19... and they have to come back and say ... because *they* started the ONE here and *they* started the ONE there … and then you might get ... Who's the little girl who got down and said that ...? … Because there is miscounting from either one or zero... because it's not a convention, it's sensible: there's the STEP, from a starting point so the one is out there ... (*gestures to the end of the step*).

Teacher 2: Then the, the ...we want the rest of the class to be standing round to watch them do that (*gestures to the circle the class would form*) ... because I had to get Denis [*pseudonym, one of the children*] to come down … and they're not used to having to look and listen to that group that's saying something important... We found when doing the project in the past that it took some lessons for them to get the idea that, 'hold on, I need to stop and listen and look to what they are saying, that has something to do with ME', and I think to get them round one of the group's number line and get them to act out what they were doing again, so that they are there seeing the number line

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> together as well, I think that's something I'd want to do in the next lesson as well.

Let us now consider this dialogue as a series of six reflective-imaginative moments in a dialectical process:

"Could you not give two different answers… And so – it could be this answer, it could be this answer …". Here the mathematical answer "19" is negated, that is, confronted directly with its opposite: the previously presumed correct mathematics is negated by an alternative answer, or alternative mathematisations of the task.

" … and actually get the children to be involved in deciding why one answer or the other". Here the children are imagined to participate and engage subjectively with the mathematical alternatives in a classroom dialogue. They are asked to reason about the mathematics with others as subjects, intersubjectively in discourse (in a debate) reflecting the opposition of the mathematical object by its mathematical negation in the measurement practice.

Researcher 1 then imagines that one of the children's (Benny's) mathematics could enter this debate somehow: the key mathematical mediation is the 'zero' reference point for the counting. This explains how the mathematically opposing objects (the counting to get 18 versus 19) become sublated in a new mathematical object-understanding (viz. where you start counting from or the counting of 'foot-prints' rather than 'foot-steps', i.e. how the task is interpreted and modelled).

Teacher 2 says "And then you might get ..." imagining now how this dialectic might come into being in her future classroom: the 'little girl' who showed, from her subjective point of view, why it is sensible to count the foot-print at the end of the first step as "one", justifying this 'correct' mathematical choice/answer.

Researcher 1 participates in the little girl's sense-making with her own gesturing, revealing why (objectifying how) the end of the step should be counted, 'one'. This subjective sharing is offered as a generalisation, and therefore as a plan for the future lesson being imagined.

Teacher 2 accepts this, and starts to envisage concretely how this could work in her future embodied teaching. She imagines how the debate might malfunction (as in past experiences) and she explains how the children need to be gathered to facilitate such a meaningful debate. This is made concrete through the recall of previous lessons where she had done this: her gestures represent the envisaged arrangement in the future class.

In previous work we have described this kind of lesson study dialogue as offering a zone of proximal development for the teachers and for profession's teaching practice, which we called a 'zone of professional development' (Radovic et al., in press). Such a dialogue can only be conceived as developmental in Vygotksyan terms if it is indeed teleological, i.e. if the teacher's professional practice is seen to be making progress towards something better. We do not know what the target professional practice is until

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after it has developed, but we can perhaps see in the dialogue the dialectic of development that our theoretical argument requires.

Arguably, then, what makes this dialogue progressive is the sublation of notions with their negations in the new imagined practices: we do not know for certain that this will be progressive until practice is confronted in the future, whereupon no doubt new contradictions will arise. Thus, professional development in teaching is a dialectical work of theory and practice; we might play with this dialectic in much the same Grundrisse-sense that Marx did with capital and labour. The classroom provides the teaching-learning practice (labour) in which professional theory (capital) is invested, and which is sublated anew in developed professional theory.

CONCLUSION

Our argument is that dialogue should be (mathematically or professionally) developmental if it is dialectical, and that the dialectic requires a concrete dialogue in which the undeveloped notion is negated in and with practice. Development then can – though of course not with certainty – arise through a genuine sublation in which the undeveloped notion and its practical negation are reconciled but both conserved in the new. The negation may involve a purely discursive, dialogical moment of negation, but at root there lies a negation in practice; its validity in developing and advancing thought is dependent on its relevance and efficacy in practice. Hegel (and Marx) describe this as sublation of the universal (general) notion in the particular, or ‘ascending from the abstract to the concrete’.

In conclusion, let us consider the implications for mathematics education and the naïve alternatives put forward in the introduction. Bakhtin’s theory of dialogism provides a rationale for the importance of the ‘internally persuasive’ dialogue with the other: thus the teachers’ and the currriculum’s mathematics must be made persuasive for effective learning to take place, and we can reject monologism that is based on arbitrary, unequal power relations. In what way can the learners’ and teachers’ mathematics be ‘equal’ in the inevitable power relations, given that the teachers’ and the curriculum’s mathematics has centuries of culturally-historically evolved science behind it?

Materialist dialectics requires that the dialogical persuasion be based in efficacy in practice: the teacher as mediator of the curriculum in practice designs or implements tasks that engage the classroom community in contradictions of given mathematical notions with practice. The teacher and curriculum can thereby arrange for dialogues in which contradictions are developmental because practice negates inadequate notions; new mathematical notions develop in which the old are sublated.

This of course applies equally to the development of the profession in lesson study processes. Well-designed lesson study should confront undeveloped theoretical ideas with challenges through classroom practice: genuine development occurs when, and because professional notions are negated (and so shown inadequate) in practice. We should perhaps celebrate and publish lesson study accounts of such inadequacies in

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classrooms much more than we usually do: they may be the life blood of real development.

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